Heisenberg's Uncertainty Principle:- It is not possible to measure simultaneously both the position and momentum of a microscopic particle with absolute accurary.

Uncertainty relation:-

Let us assume that mean squere deviation is related to uncertainty.

Consider A is the Hermitian operator.

$$A = A^{+}$$

Let

$$A = A - \langle A \rangle$$

$$\langle A \rangle = \langle A - \langle A \rangle \rangle$$

$$\langle A \rangle = \langle A \rangle - \langle A \rangle \qquad (\because \langle \langle A \rangle \rangle = \langle A \rangle)$$

$$\langle A \rangle = 0$$

$$\langle A^2 \rangle = \langle (A - \langle A \rangle)^2 \rangle$$

$$\langle A^2 \rangle = \langle (A^2 + \langle A \rangle^2 - 2A\langle A \rangle) \rangle$$

$$\langle A^2 \rangle = \langle A^2 \rangle + \langle A \rangle^2 - 2\langle A \rangle \langle A \rangle$$

$$\langle A^2 \rangle = \langle A^2 \rangle + \langle A \rangle^2 - 2\langle A \rangle^2$$

$$\langle A^2 \rangle = \langle A^2 \rangle - \langle A \rangle^2 \quad \text{--- (1)}$$

$$\langle A^2 \rangle = (\Delta A)^2$$

uncertainty in A is $(\Delta A)^2 = \langle A^2 \rangle$ --- (A)

uncertainty in B is $(\Delta B)^2 = \langle B^2 \rangle$ --- (B)

MJC-8 .

$$A = A - \langle A \rangle$$

$$B = B - \langle B \rangle$$

A and B are the properties of uncertainty

Let us assume another operator which is not Hermitian.

$$O = A + i\lambda B$$

$$O^{+} = A - i\lambda B \qquad (\because O \neq O^{+})$$

assume system is stated by ψ

$$O\psi = \phi \quad \text{——} \ (2)$$

$$\langle A \rangle = (\phi A \phi) = \int \phi^{*} A \phi \, d\tau \geq 0$$

$$(O\psi, O\psi) = (\psi, O^{+}O\psi)$$

$$(O\psi, O\psi) = \psi, (A - i\lambda B)(A + i\lambda B)\psi) \quad \text{——} \ (3)$$

$$\langle A \rangle = (\psi, A\psi)$$

$$\langle (A - i\lambda B)(A + i\lambda B) \rangle \geq 0$$

$$\langle A^2 + \lambda^2 B^2 + i\lambda (AB - BA) \rangle \geq 0$$

$$\langle A^2 + \lambda^2 B^2 + i\lambda [A, B] \rangle \geq 0$$

$$\underbrace{\langle A^2 \rangle + \lambda^2 \langle B^2 \rangle + i\lambda \langle [A, B] \rangle}_{f(\lambda)} \geq 0 \quad \text{——} \ (4)$$

MJ C-8.

$$f(\alpha) = \langle A^2\rangle + \alpha^2\langle B^2\rangle + i\alpha\langle [A, B]\rangle$$

$f(\alpha)$ should be small as possible

$$\frac{df(\alpha)}{d\alpha} = 0$$

$$2\alpha\langle B^2\rangle + i\langle [A, B]\rangle = 0$$

$$\alpha = -\frac{i}{2\langle B^2\rangle}\langle [A, B]\rangle$$

Put the value of α in equation (4).

$$\langle A^2\rangle - \frac{\langle [A, B]\rangle^2 \cancel{\langle B^2\rangle}}{4\langle B^2\rangle^{\cancel{2}}} + \frac{\langle [A, B]\rangle^2}{2\langle B^2\rangle} \geq 0$$

$$\langle A^2\rangle - \frac{\langle [A, B]\rangle^2}{4\langle B^2\rangle} + \frac{2\langle [A, B]\rangle^2}{4\langle B^2\rangle} \geq 0$$

$$\langle A^2\rangle + \frac{\langle [A, B]\rangle^2}{4\langle B^2\rangle} \geq 0$$

$$\langle A^2\rangle \geq -\frac{\langle [A, B]\rangle^2}{4\langle B^2\rangle}$$

$$\langle A^2\rangle\langle B^2\rangle \geq -\frac{\langle [A, B]\rangle^2}{4}$$

$$\boxed{\langle \Delta A)^2(\Delta B)^2 \geq -\frac{\langle [A, B]\rangle^2}{4}}$$ from equation (A) and (B)

let $A = x$ position

$B = p_x$ Momentum, $p_x = -i\hbar\frac{\partial}{\partial x}$

$$(\Delta x)^2 (\Delta P_x)^2 \geq - \frac{\langle [x, P_x] \rangle^2}{4}$$

$$[x, P_x] = -i\hbar$$

$$(\Delta x)^2 (\Delta P_x)^2 \geq - \frac{(\langle -i\hbar \rangle)^2}{4}$$

$$(\Delta x)^2 (\Delta P_x)^2 \geq - \frac{i^2 \hbar^2}{4} \qquad (i^2 = -1)$$

$$(\Delta x)^2 (\Delta P_x)^2 \geq \frac{\hbar^2}{4}$$

$$\boxed{\Delta x \, \Delta P_x \geq \frac{\hbar}{2}} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$h \to$ Planck's constant

This is called uncertainty principle.

If Δx is small then ΔP_x will be very large. (uncertainty in momentum)

If ΔP_x is small then Δx is very large (uncertainty in position).

we cannot see a microscopic particle without disturbing it.

$$\Delta y \, \Delta P_y \geq \frac{\hbar}{2}, \qquad \Delta z \, \Delta P_z \geq \frac{\hbar}{2}.$$

or

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}.$$